

## ANNOTATED BIBLIOGRAPHY TRANSFORMATIONAL GEOMETRY TASK FORCE

### Books & Articles

- Battista, M. T. (2007). The development of geometric and spatial thinking. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843-908). Reston, VA: National Council of Teachers of Mathematics.
- Bellomo, C. (2007). A treasure hunt: Reflecting, translating, and rotating points on a coordinate map. *Mathematics Teaching in the Middle School* 13(5): 316-20.  
[http://www.nctm.org/eresources/article\\_summary.asp?URI=MTMS2007-12-316a&from=B](http://www.nctm.org/eresources/article_summary.asp?URI=MTMS2007-12-316a&from=B)
- Bennett, D. (2002). *Exploring geometry with the Geometer's Sketchpad* (pp. 38-39, 125-128). Emeryville, CA: Key Curriculum Press.  
Presents four activities in which students use Geometer's Sketchpad to investigate what happens to coordinates when reflected in a line (p. 38), translated (p. 39 and pp. 125-126), and rotated (pp. 127-128).  
<http://www.keypress.com/x5581.xml>
- Bussi, M.B. & Boero, P. (1998). Teaching and learning geometry in contexts. In C. Mammana & V. Valani (Eds.), *Perspectives on the teaching of geometry for the 21<sup>st</sup> century*. (pp.). Boston: Kluwer Academic Publishers.
- Clemens, D.H. (2003). Teaching and learning geometry. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 151-178). Reston, VA: National Council of Teachers of Mathematics.  
This  
<http://www.nctm.org/catalog/product.aspx?ID=12341>
- Crowley, M.L. (1987). The van Hiele Model of the development of geometric thought. In M.M. Lindquist (Ed.), *Learning and teaching geometry, K-12* ( pp. 1-16). Reston, VA: National Council of Teachers of Mathematics.  
An accessible introduction to the van Hiele Model including suggested activities to help students move through the levels of development.
- Day, R., Kelley, P., Krussel, L., Lott, J.W., & Hirstein, J. (2002). *Navigating with geometry for grades 9-12*. Reston, VA: National Council of Teachers of Mathematics.  
Includes a chapter on isometries and a chapter on dilations.  
<http://www.nctm.org/catalog/product.aspx?ID=12175>

- Driscoll, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Portsmouth, NH: Heinemann.
- A book for use with teachers in grades 5-10. Chapter 3 is a great introduction to the issues surrounding the teaching of transformational geometry particularly at the middle grades. It puts appropriate stress on the fact that transformations are functions of all points in the plane to points in the plane. Includes a CD. There is also an associated professional development toolkit that is very expensive and less useful than the book.
- <http://www.heinemann.com/products/E01148.aspx>
- Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Valani (Eds.), *Perspectives on the teaching of geometry for the 21<sup>st</sup> century*. (pp. 37-52). Boston: Kluwer Academic Publishers.
- Faulkenberry, E.D., & Faulkenberry, T.J. (2010). Transforming the way we teach functional transformations. *Mathematics Teacher* 104(1): 29-33.
- This article describes using the input-output definition of functions and transparencies to analyze transformations of functions.
- [http://www.nctm.org/eresources/article\\_summary.asp?URI=MT2010-08-29a&from=B](http://www.nctm.org/eresources/article_summary.asp?URI=MT2010-08-29a&from=B)
- Fuys, D., Geddes, D., & Tischler, R. (1988). *The van Hiele model of thinking in geometry among adolescents*. Reston, VA: National Council of Teachers of Mathematics.
- A report of the original research done by the team that had the van Hiele's materials translated into English.
- Glass, B. (2004). Transformations and technology: What path to follow? *Mathematics Teaching in the Middle School* 9(7): 392-97.
- [http://www.nctm.org/eresources/article\\_summary.asp?URI=MTMS2004-03-392a&from=B](http://www.nctm.org/eresources/article_summary.asp?URI=MTMS2004-03-392a&from=B)
- Herbst, P.G. (2006). Teaching geometry with problems: Negotiating instructional situations and mathematical tasks. *Journal for Research in Mathematics Education* 28(5): 524-49.
- Hollebrands, K.F. (2007). The role of a dynamic software program for geometry in the strategies high school students employ. *Journal for Research in Mathematics Education* 38(2): 164-192.
- [http://www.nctm.org/eresources/article\\_summary.asp?from=B&uri=JRME2007-03-164a](http://www.nctm.org/eresources/article_summary.asp?from=B&uri=JRME2007-03-164a)
- Hollebrands, K.F. (2004). High school students' intuitive understandings of geometric transformations. *Mathematics Teacher* 97(4): 207-214.

[http://www.nctm.org/eresources/article\\_summary.asp?URI=MT2004-03-207a&from=B](http://www.nctm.org/eresources/article_summary.asp?URI=MT2004-03-207a&from=B)

Jacobsen, C. & Lehrer, R. (2000). Teacher appropriation and student learning of geometry through design. *Journal for Research in Mathematics Education* 31(1): 71-88.

[http://www.nctm.org/eresources/article\\_summary.asp?from=B&uri=JRME2000-01-71a](http://www.nctm.org/eresources/article_summary.asp?from=B&uri=JRME2000-01-71a)

Johnston-Wilder, S. & Mason, J. (Eds.). (2005). *Developing thinking in geometry*. London: Paul Chapman Publishing.

Text that includes background discussion excellent for professional developers and teachers. See especially chapter 9-12 that focus on transformations. Includes a CD.

<http://www.uk.sagepub.com/books/Book228127>

Laborde, C., Kynigos, C., Hollebrands, K. & Strässer, R. (2006). Teaching and learning geometry with technology. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 275-304). Rotterdam, The Netherlands: Sense Publishers.

<http://www.sensepublishers.com/catalog/files/90-77874-19-4.pdf>

Lehrer, R. & Chazan, D. (1998). *Designing learning environments for developing understanding of geometry and space*. London: Psychology Press.

Marrades, R. & Gutiérrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics* 44(1-3): 87-125.

Mayberry, J. The van Hiele levels of geometric thought in undergraduate preservice teachers. *Journal for Research in Mathematics Education* 14(1):58-69.

Pugalee, D.K., Frykholm, J., Johnson, A., Slovin, H., Malloy, C., & Preston, R.. (2002). *Navigating through geometry in grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

O'Daffer, P.G, & Clemens, S.R. (1992). *Geometry: An investigative approach 2<sup>nd</sup> edition*. Place?: Pearson.

Serra, M. (1994). *Patty paper geometry: Student workbook*. Emeryville, CA: Key Curriculum Press.

Shockey, T.L. & Snyder, K. (2007). Engaging preservice teachers and elementary-age children in transformational geometry: Tessellating T-shirts. *Teaching Children Mathematics* 14(2): 82.

[http://www.nctm.org/eresources/article\\_summary.asp?from=B&uri=TCM2007-09-82a](http://www.nctm.org/eresources/article_summary.asp?from=B&uri=TCM2007-09-82a)

Slovin, H. (2000). Moving to proportional reasoning. *Mathematics Teaching in the Middle School* 6(1): 58-61.

[http://www.nctm.org/eresources/article\\_summary.asp?from=B&uri=MTMS2000-09-58a](http://www.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2000-09-58a)

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades 5-8* (pp. 216-219). Boston: Pearson Education.

Presents a discussion and a series of introductory activities investigating the effect on transformations on coordinates.

Yaglom, I.M. (1962). *Geometric transformations, I, II, III*. Washington, DC: Mathematical Association of America.

Yanik, H.B. & Flores, A. (2009). Understanding rigid transformations: Jeff's learning path for translation. *The Journal of Mathematical Behavior* 12(1): 41-57.

### **Electronic**

Addington, S.

Applets for doing dilations using GeoGebra of triangles, quadrilaterals, and pentagons.

<http://www.quadrivium.info/GGB/SimTri.html>

<http://www.quadrivium.info/GGB/SimQuad.html>

<http://www.quadrivium.info/GGB/SimPenta.html>

Cornrow Curves.

Teaching materials for using cornrow curves as a cultural context for transformations.

[http://csdt.rpi.edu/african/cornrow\\_curves/teaching/teaching.html](http://csdt.rpi.edu/african/cornrow_curves/teaching/teaching.html)

Covering the Plane with Rep-Tiles.

This lesson for middle school students involves rep-tiles. Rep-tiles are geometric figures such that  $n$  copies can fit together to form a larger, similar figure. Students create tessellations using rep-tiles.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L251>

Dilations.

This applet allows one to change the scale factor and center point including negative scale factors.

<http://www.waldomaths.com/Enlarge1NLW.jsp>

Geometric-thinking.org Dilations Applet.

Provides an applet to explore how changing the center of a dilation changes the image of a given triangle. One can also change the scale factor.

<http://www.geometric-thinking.org/dilations.htm>

Inversions.

This lesson for middle school students explores the symmetry of letters of the alphabet.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L882>

Math Bits.

Uses Geometer's Sketchpad 4.0 to do dilations.

<http://mathbits.com/mathbits/gsp/wkdilate.pdf>

Math Forum Mathtools.

This "Math Tools" webpage provides a number of computer-based tools and lessons focusing on Transformations.

<http://mathforum.org/mathtools/cell.html?co=g&tp=10.13&sortBy=&offset=75&limit=25>

Math Is Fun.

<http://www.mathsisfun.com/geometry/translation.html>

Math Open Reference.

This site has various subsites that are instructional in nature but also include some dynamic ability to change images and see, for example, the change in scale factor.

<http://www.mathopenref.com/tocs/similaritytoc.html>

Music and Transformations.

Explains isometries using music.

<http://www.squarecirclez.com/blog/music-and-transformation-geometry/5074>

Stevens, Cullen. Dilations.

[http://www.geogebra.org/en/upload/files/english/Cullen\\_Stevens/dilations\\_scalefactor.html](http://www.geogebra.org/en/upload/files/english/Cullen_Stevens/dilations_scalefactor.html)

Symmetries I.

This NCTM Illuminations webpage, Symmetries II, is part of a series titled "Symmetries and their Properties." This unit explores rotations.

<http://illuminations.nctm.org/LessonDetail.aspx?ID=U138>

#### Symmetries II.

This NCTM Illuminations webpage, Symmetries II, is part of a series titled “Symmetries and their Properties.” This unit explores reflections.

<http://illuminations.nctm.org/LessonDetail.aspx?ID=U139>

#### Translation Lesson Using GeoGebra.

[http://www.geogebra.org/en/upload/files/english/Guy/Transformations/Translation\\_Lesson.pdf](http://www.geogebra.org/en/upload/files/english/Guy/Transformations/Translation_Lesson.pdf)

#### Transformation Golf.

This site has a golf game format to practice rotations, reflections and translations.

<http://www.mathsonline.co.uk/nonmembers/gamesroom/transform/golfrans.html>

#### Transformations.

A whole unit on isometries and dilations that includes how to do each with patty paper.

[http://online.math.uh.edu/MiddleSchool/Modules/Module\\_4\\_Geometry\\_Spatial/Content/TransformationsTEXT.pdf](http://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Content/TransformationsTEXT.pdf)

#### Understanding Congruence, Similarity, and Symmetry Using Transformations and Interactive Figures.

This NCTM Illuminations webpage provides a unit on transformations that includes interactive applets. The unit is titled “Understanding Congruence, Similarity, and Symmetry Using Transformations and Interactive Figures.”

<http://illuminations.nctm.org/LessonDetail.aspx?ID=U134>

#### Utah State University Online Math Lessons.

This Utah State University website provides a series of interactive web-based lessons on reflections, translations, rotations, and dilations.

<http://enlvm.usu.edu/ma/nav/toc.jsp?sid=shared&cid=emready@transformations&cf=activity>

### **Software**

Geometer’s Sketchpad: Dynamic Geometry Software for Exploring Mathematics version 5

GeoGebra: free download