Professional Development*
for
Transformational Geometry
Reflections

**This is written as a multi-day investigative lesson that can be taught in the classroom. This means that participants in the Professional Development workshop would be students in the classroom. Consequently, the notes given, and suggested questions are encouraged to be modeled by the PD provider with the understanding that they are modeling what the teachers might state or ask in their own classrooms. There fore it is recommended either at the beginning or end of this unit participants are given a full copy of this lesson, both the teacher pages and the resource pages.*

The PD provider will need to use their professional judgment on how to 'tweak' this to meet their audience's needs. Eighth grade teachers may need additional scaffolding on the mathematics and high school teachers may need more assistance with questioning and pedagogy.

Objective

Participants will use their spatial visualization skills to investigate reflections. Participants will understand the connections between an original figure and its image** and make summary statements on their findings.

***image: the resulting point or set of points under a transformation*

Materials (one per participant)

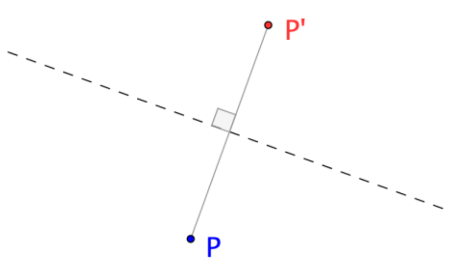
- Graph Paper
- Colored Pencils (no light colors)
- Rulers
- Tracing Paper (optional)
- Miras or colored CD cases (if technology is not available)

Resource Page(s) (one per participant)

- Exploring Reflections
- Reflections: What Can I Figure Out About the Reflection of an Image?
- Reflections: Making Sense of the Coordinates
- Reflections: More Practice
- Reflections: Challenge

Reflections

A <u>reflection</u> maps every point P to a point P' such that: (1) if P is not on the axis of reflection then the axis is the perpendicular bisector of $\overline{PP'}$ (the line segment joining P and P'), and (2) if P is <u>on</u> the axis of reflection, then $P = P'$.

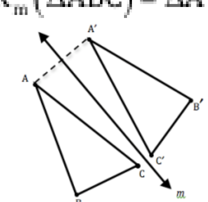


Properties

More loosely stated, a reflection is a "flip" over a reflection line such that:

- For any point not on line ***m***, ***m*** is the perpendicular bisector of a point on the original figure and its corresponding reflected point, therefore ***m*** is the perpendicular bisector of $\overline{AA'}$ where A' is the image of **A.** This holds true for all points. We will write $R_m(\Delta ABC) = \Delta A'B'C'$

Representation



<table>
<tr><td>Website(s):
• www.mathsisfun.com/geometry/reflection</td><td>(Suggested) Study Team Strategies:
Pairs Check</td></tr>
</table>

Standards Addressed
(The specific part of the standard addressed is bolded. Consider having participants highlight this themselves after the lesson is over).

8th Grade Common Core	High School Geometry Conceptual Category
Geometry 8.G	**Congruence G-CO**
Understand congruence and similarity using physical models, transparencies, or geometry software. 1. *Verify* ***experimentally*** *the properties of rotations,* ***reflections****, and translations.* ***a. Lines are taken to lines, and line segments are taken to line segments of the same length.*** *b. Angles are taken to angles of the same measure.* ***c. Parallel lines are taken to parallel lines.*** *3. Describe the effects of dilations, translations, rotations and* ***reflections on two-dimensional figures using coordinates.***	**Experiment with transformations in the plane** *2.* ***Represent transformations in the plane using, e.g. transparencies and geometry software;*** *describe transformations as* ***functions that take points in the as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not*** *(e.g. translations versus horizontal stretch).* ***3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the*** *rotations and* ***reflections that carry it onto itself.*** ***4. Develop definitions of*** *rotations,* ***reflections,*** *and translations in terms of angles, circles,* ***perpendicular lines, parallel lines, and line segments.*** 5. ***Given a geometric figure*** *and a rotation,* ***reflection****, or translation, draw the transformed figure using e.g.,* ***graph paper****, tracing paper, or geometry software. Specify a sequence of transformations that will carry* ***a given figure onto another.***

At the end of this mini unit, have participants look at the three Standards for Mathematical Practice (see next page) and identify which parts of each were addressed and specifically how they were addressed. This could be done as a "Think-ink-Pair-share" or as a dyad followed by a whole class discussion.

Mathematical Practices

4 Model with mathematics.
Mathematically proficient participants can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a participant might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a participant might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient participants who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

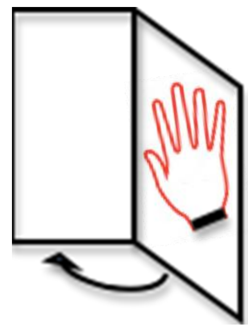
5 Use appropriate tools strategically.
Mathematically proficient participants consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient participants are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school participants analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient participants at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.
Mathematically proficient participants look closely to discern a pattern or structure. Young participants, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, participants will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older participants can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Activity Overview (in a classroom this would take place over several days)

Part I: Reflecting Your Hand

Start by asking participants, *"If you held your right hand up to a mirror, what would you see?* (Answers might include, *"my hand",* or *"a reflection of my hand"…*) One or more participants may demonstrate what they see by holding their left hand up such that it mirrors the right hand. If they don't, suggest that they hold their left hand such that it represents the mirror's image of your right hand. Next have participants fold their piece of graph paper 'like a hamburger' and trace their right hand on the right side using a colored pencil. This needs to be really dark, so after they remove their hand they most likely will need to darken the line by going back and forth over it with their color pencil a section at a time. Next ask participants to add a bracelet or ring to their hand, making sure that opposite sides are parallel.



Instruct participants to refold their paper and rub their nail along the pencil line to make a tracing of their hand on the left side of the paper.

Ask participants "*What mathematical relationships can you discover between the drawing of your right hand, the tracing on the left and the fold line? Mark these on your paper."* If participants do not automatically draw lines between corresponding parts of their original tracing and its image, ask guiding questions such as, *"Can you pick two corresponding points, one from each hand, and describe the relationship between them? Does that relationship hold true for and two corresponding points?"*

After a brief whole class discussion, pass out the resource page **Exploring Reflections** and ask participants to write a *summary statement* using complete sentences in the top box labeled **Hand Activity.** Modeling what you would do in the classroom, ask several participants to share out to make sure summaries are complete/correct and ask participants to make changes as needed.

Participant statements should include the ideas that the fold line becomes the perpendicular bisector between two corresponding points on their hand and its image and the parallel lines from the ring or bracelet are still parallel.

Part II: Technology

Direct participants to the site: www.mathisfun.com/ geometry/reflection/ and to read the introduction (above the graph) which should confirm what they already wrote on their resource page. You might pose the question, *"Does this seem to match what we did with our hand activity? How so?"* to make sure the participants are making the connection between the two activities. Direct participants to the lower left hand corner of the graph to click on **View Larger** without reading the summary of information below it; this is something it would be best for participants to explore and come up with on their own and/or with teacher guidance as necessary. Have participants complete one or more reflections across each line. As you circulate ask questions such as, *"Can you predict exactly where the image will be on the graph?" How can you find the location?"* If participants do not come up with the idea of comparing coordinates between the original and reflected point, then you may need to be more specific and ask them to name a point and its corresponding point using coordinates. A whole class discussion may be necessary. Participants should record their observations as they work on the lower half of the **Exploring Reflections** resource page in the box marked **Observations**. Have several participants share out their observations and then consolidate the ideas stated as a closure for these two activities.

<u>If Technology is limited</u>

If only a teacher computer is available with projection capabilities, then this can be done as a whole class activity. Consider having participants sketch the axis and the original figure then draw in their prediction of the reflected image on their papers. A **pair check** could quickly be done before the teacher clicks the reflect button to show the actual results. Participants could then come up to the board and demonstrate the relationship between the coordinates on the coordinates on the original figure and the reflected figure.

<u>If Technology is not available</u>

If technology is not available, the above activity can be done either with Miras or colored CD case, if Miras are not available. Either provide participants with figures to be reflected on a handout or have them draw their own. If participants are asked to draw their own, encourage them to use simple shapes and to line up vertices on lattice points to easily read their coordinates and that of the image. Have participants then place the Mira of CD case on the line of reflection and carefully draw the reflected image on the other side of the line.

<u>Part III: Reflections:</u> What Can I Figure Out About the Reflection of an Image?

If this is **Day 2** for this lesson, start off by having a class discussion on the previous day's findings.

Pass out the resource page **<u>Reflections:</u> What Can I Figure Out About the Reflection of an Image?** to participants and discuss the directions with them and then have them start on parts a) and b). As you circulate, make sure participants notice the scale (lines are one unit apart), and they are putting coordinates on the vertices of both the original and reflected image, if not, remind them to do so. They will need this information for their summary statement.

Note: If students are struggling with drawing the reflections, remind them that they can darken the original figure then fold their paper across the line of reflection and then rub to make a tracing to get started.

Pull the class together after they have completed parts a) and b). Have some of the participants read their statements. Have participants make corrections and/or additions as necessary. Discuss the mathematical notation and make sure they have added that to their summary statements. A possible summary statement for part a) might be:

"All of the x-coordinates stay the same but the y-coordinates switch signs. The x-axis is the perpendicular bisector of each line connecting the corresponding points on the original figure and it's reflected image".

Direct participants to read and start part c). After they have labeled the four quadrants for parts a) and b), pose the question, *"Is there a relationship between these signs and what happens to the coordinates when you reflect across the x- and y- axis? "* Give some wait time before you solicit responses. Participants should understand, and, complete part c) before they move on.

Make sure participants recognize the significance of the equation of the line *y = x in part d).* They, the participants' students, may not recognize that it literally means that *y = x* and *x = y.* Recognizing this should help them predict the coordinates of the reflected image in part d). A possible summary statement might be:

"All of the x- and y-coordinates traded places. Like the equation said y = x and this means that x also equals y. It is almost like the figure flipped over the y-axis and then over the x-axis".

Part e) should be done as a whole class discussion. After participants have reflected the rectangle across the line ***y=x***, pose the question stated on the resource page: "*How do you think reflecting the rectangle affects the other figures in Plane A? Write down your ideas in the space provided*." Have participants share out some of their ideas before saying something like:

> *"Although we have not talked about this idea explicitly, every reflection affects the entire plane—not just a single figure. So if we reflect the rectangle about any line, all other figures, in fact, every point within the plane, are also reflected. To illustrate this point, revisit your graphs in parts b), d) and e) and add in the necessary images based upon this new information".*

Part f) is a pre-problem for the next day's lesson. Students are directed to reflect their new images from part e) about the x-axis. Discuss if there is a different transformation they could use to get from the original figures to the figures from the two reflections. Students may see that a 90° Clockwise rotation about the origin would accomplish this. Close the day or this portion of the day with a summary discussion with participants.

Part IV: Reflections: Making Sense of Coordinates

If this is the start of **Day 3** of the lesson, start with a whole class discussion on the findings from the last two days' activities, **Exploring Reflections** and **Reflections: Summarizing Your Findings,** parts a) and b). This is not meant to be a re-teaching, but rather an effort to re-focus participants' (remember, students') thinking for today's tasks…

Today participants will investigate the effect that reflecting a figure about the lines ***y = x*** and ***y = -x*** has on the coordinates of the reflected image. Participants will organize their information into tables provided on the resource pages and will be asked to explain their thinking. Guiding questions for part a) might include, *"What information is the line **y=x** telling us about the coordinates of the reflected image? How can that be used to help you graph this image?"* For some students, shading in the two figures may be visually useful. You might also ask, "*If I were to connect corresponding points in the two triangles, what would be relationship between the line of reflection and these lines?".* If they do not readily remember the line of reflection will be the perpendicular bisector of each of these segments, you might ask them to draw these lines in to remind themselves.

Part b) asks participants to reflect the triangle across the line ***y=-x****;* there are also two parallel line segments of equal length that should also be reflected since they are in the same plane. As participants finish this page, bring them together for a whole class discussion to consolidate ideas. This should include the relationship of the original figure and its image when reflected across the lines ***y=x*** and ***y=-x***:

> *parallel lines remain parallel; length is preserved and the reflection line is the perpendicular bisector of line segments connecting corresponding points.*

Parts c) and d) look at a composition of reflections. Participants should see that:

> *the effect of reflecting across two different lines is a translation if the lines are parallel and a rotation if they are not parallel.*

Patty paper can be useful to verify this information. (Note: Every 2-dimensional isometry is the product of at most three reflections).

Part V: Reflections: Challenge

This is the concluding activity of this multi-day lesson or mini-unit. Participants are challenged to make the minimum number of reflections to 'find their way back home' without undoing a previous move. This leads into other transformations, rotation and translation. Patty paper can be used to demonstrate or explore these transformations. At this point, the concepts of isometries could be introduced and the requirement that size, and shape must be preserved.

Part VI: Formative Assessment

The formative assessment asks for participants to create a stand-alone poster to consolidate their understanding of reflections. This would best be done in teams of 2-4 participants. After posters are complete, they can be posted up on the wall for a gallery walk. Participants should be encouraged to take notes on other posters if they missed some of the connections. After the gallery walk consider giving time for teams to revisit their own poster to make additions or corrections based upon their notes.