VISUALIZING ROTATIONS
Professional Development Notes

Summary	**Goals**	**Participant Pages**
Participants experiment with rotations, and learn some properties of this isometry. Participants use rotations to create a design. Participants explore how translations and rotations can be obtained from a composition of reflections.	• Distinguish between a translation, reflection, and rotation. • Visualize, and then perform rotations using patty paper. • Apply properties of rotations to create a design. • Explore how translations and rotations can be obtained from the composition of reflections.	PP1: Visualizing Rotations PP2-3:Rotations PP4: Practice with Rotations PP5: Rotation Design Project PP6: Exploring Definitions
Reproducibles	**Materials**	**Technology**
Reproduce standards page and word bank for participants if desired.	• Patty paper (2-3 sheets per participant) • Colored pencils • internet access (for demonstration) • protractor • circular geoboard (optional) • polar graph paper (optional)	The rotation introduction will be enhanced with http://www.waldomaths.com/ At this website go to Ages 11 – 16, and then rotations.

MATHEMATICS STANDARDS

COMMON CORE STATE STANDARDS FOR MATHEMATICS

8.G.1a	Verify experimentally the properties of rotations, reflections, and translations: Lines are taken to lines, and line segments to line segments of the same length.
8.G.1b	Verify experimentally the properties of rotations, reflections, and translations: Angles are taken to angles of the same measure.
8.G.1c	Verify experimentally the properties of rotations, reflections, and translations: Parallel lines are taken to parallel lines.
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
G-CO-2	Represent transformations in the plane using, e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation vs. horizontal stretch).
G-CO-3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G-CO-4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G-CO-5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry the given figure onto another.

STANDARDS FOR MATHEMATICAL PRACTICE

Some of the Standards for Mathematical Practice that are applied in this lesson are:

3 Construct viable arguments and critique the reasoning of others.

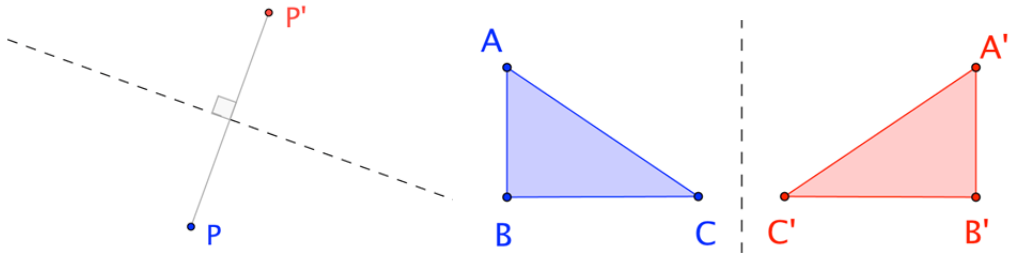
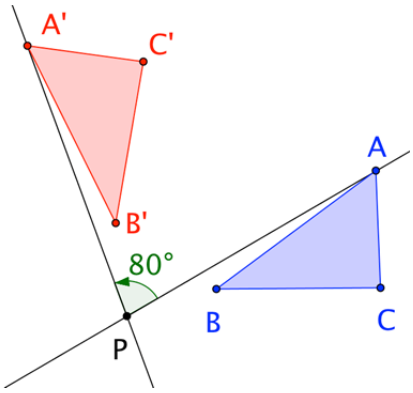
Participants make a conjecture about the relationship between the distance from the center of a rotation and corresponding points in the figure and its image. They refine the conjecture as needed with additional data.

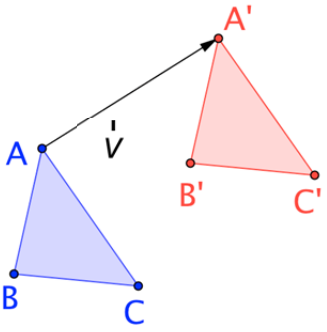
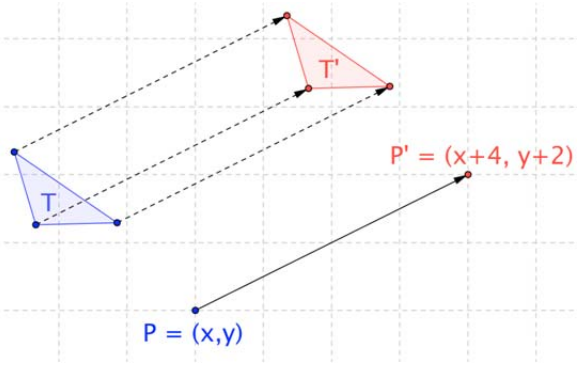
5 Use appropriate tools strategically.

Tools that emphasize how transformations work dynamically help participants to develop meaning for the concept. Tools used here include patty paper, protractor, and an interactive applet available on the internet.

7 Look for and make use of structure.

Through examples, participants will begin to generalize properties of transformations. They will observe that isometries preserve lengths, parallelism, and angle measure. They will observe that the center point of the rotation is invariant, and that all other points map to another point in the plane through the defined transformation.

WORD BANK	
isometry	An isometry is a transformation of the plane that preserves length.
image	An image is the resulting point or set of points under a mapping (function).
reflection	A reflection maps every point P to a point P' such that: (1) if P is not on the axis of reflection then the axis is the perpendicular bisector of $\overline{PP'}$ (the line segment joining P and P'), and (2) if P is on the axis of reflection, then $P = P'$. 
rotation	A rotation about a point P through angle α is a transformation such that: (1) if point A is different from P, then $PA = PA'$ and the measure of $\angle APA' = \alpha$; and (2) if point A is the same as P, then $A' = A$. 

WORD BANK	
translation	A <u>translation</u> of the plane shifts all points on the plane in the same direction and in the same distance. That is, given a vector (directed segment) $\vec{v}$, the image P' of a point P is the point for which $\overline{PP'}$ is is parallel to to $\vec{v}$ and $PP' = \left\|\vec{v}\right\|$.  
transformation	A <u>transformation</u> of the plane is a one-to-one mapping (function) of the plane onto itself.

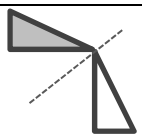
WARMUP

Whole Class

- SP1 Visualizing Rotations

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.

- Review translations (Slides), and reflections (Flips) and introduce rotations (Turns). Compare transformations and discuss.

		Transformation S	Transformation F	Transformation T
1	figure			
2	steps	Slide figure to right a length equal to the bottom leg of the triangle	Flip figure along the dotted line	Rotate figure 90^{o} in a counter clockwise direction
3	congruent	All figures in this warmup and their images in this warmup are intended to be congruent. Review basic properties of isometries: distance, parallelism, and angle measure.		
4	orientation	Same	Different	Same
		Orientation can be checked by observing the sequence of lettering around a polygon. Transformations with the same orientation have lettering in the same direction (clockwise). Only reflections reverse order. Rotations turn, but the orientation remains the same		
5	name	Translation "slide"	Reflection "flip"	Rotation "turn"

INTRODUCE

Whole Class

- To review angle measures (especially multiples of 90^o), invite some participants to stand in a circle at the front of the room, and choose a participant or an object as the center. Have participants move, as a group, in multiples of 90^o around the center. Participants at their seats can show movement in the air with their finger. Emphasize that movement in a counterclockwise direction is positive. Movement in a clockwise direction is negative.

➢ SP2 Rotations

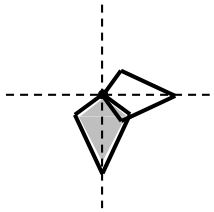
Use the internet for a rotation demonstration in conjunction with this activity.
Go to http://www.waldomaths.com/Rotations1NLW.jsp

Materials:
colored pencils
patty paper
internet access

- Work the first problem as a group. Have participants imagine where a 90^o rotation of the plane will leave an image of the kite.

- Distribute at least one piece of patty paper (or tracing paper) to each participant. Ask participants to draw the kite figure on patty paper with heavy pencil markings on both sides. This will make the back of the patty paper act as carbon paper to transfer the image back to the participant page.

- Participants put patty paper image on top of the figure, put the point of their pencil on the center point of the rotation, and rotate the degrees indicated. Then they trace over the result. The image should transfer.



- Ask questions about the rotation transformation.

Under the rotation, are there any points that do not move? Yes, the center point stays invariant.

What do you observe the relationship segments formed by corresponding points and the center? They are equal to each other.

Is distance preserved? What does that mean? Yes. All lengths of segments remain the same. Lines are taken to lines, and line segments to line segments of the same length.

Is parallelism preserved? What does this mean? In general, yes. Parallel lines are taken to parallel lines. In this figure, there are no parallel lines. This property can be demonstrated by drawing a line segment parallel to one of the sides of the kite and observing its image under the rotation. Since the rotation is a transformation of the plane, the added line segment will move along with the figure.

Are angle measures preserved? Yes. What does this mean? Angles are taken to angles of the same measure.

EXPLORE

Groups

- SP3 Rotations (continued)

Materials:
Patty paper
Colored pencils
Internet access

- Participants use patty paper and internet applet (if available) to visualize and rotate objects in the plane. Encourage them to predict before rotating.

SUMMARIZE

Whole Class

- SP3 Introduction to Isometries

- Ask participants to state and explain the meaning of some of the key properties of rotations. Record them on the board, and revise language for clarity as a group. Some possible statements are:

  Rotations preserve distance.
  Rotations preserve parallelism.
  Rotations preserve angle measure.
  Rotations maintain orientation.
  Rotations preserve collinearity.
  Rotations preserve betweenness.
  A <u>rotation</u> is a transformation such that the image of every point is a specified angle from a fixed point, called the center point of the rotation. (definition)
  A <u>rotation</u> is a transformation such that the image of every point is a specified angle from a fixed point, called the center point of the rotation. a fixed point, called the center point of the rotation. (this is the conjecture from the participant pages)

PRACTICE

Individuals

- SP4 Practice with Rotations

This page is appropriate for practice or homework. Alert participants that in the first group they will rotate a figure to its image. In the second group, they will "undo" a rotation operation. The image is given and they need to find an appropriate initial figure.

EXTEND 1

Individuals

- SP5 Rotation Design Project

This project will give participants an opportunity to apply some properties of rotations. In this project, the initial angle of rotation is 60^{o}. Encourage participants to try rotations of other degrees and to change the figure under rotation. A circular geoboard or polar graph paper will help participants locate equally spaced points on a circle if they wish to try rotations of other degrees.

EXTEND 2	
Whole Class/Groups ➢ SP6 Exploring Definitions	• Revisit the definitions for translations and rotations, which are based on composition of reflections. Invite participants to explore these definitions further to convince themselves that they are equivalent to the explanations and interpretations developed informally throughout the lesson.
CLOSURE	
Whole class ➢ SP1 Visualizing Rotations	Review standards, vocabulary, and goals for lesson.