Investigating Transformations Using Coordinates

Overview

This is a series of explorations that could be used as a single lesson or used as smaller lessons interspersed throughout a unit on transformations.

Pre-Requisites	Basic knowledge of rotations, reflections, and translations (8.G.1)
Standards Addressed	8.G.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Mathematical Practice(s)	MP 2: Reason abstractly and quantitatively MP 3: Construct viable arguments and critique the reasoning of others MP 7: Look for and make use of structure
Handouts	Investigating Transformations Using Coordinates –page 1 (4 copies per student) Investigating Dilations Using Coordinates– *page 2-3* (1 copy per student) Transformations Using Coordinates Summary – *page 4* (1 copy per student)
Overview	**Key Question:** ▪ How do the different transformations – dilations, translations, rotations, and reflections – affect the coordinates of two-dimensional figures? **Part 1: Translations** *(pages 2-3)* **Part 1: Reflections** *(pages 4-7)* **A. Reflections in the x-axis and y-axis** **B. Reflections in lines (not axes)** **Part 2: Rotations** *(pages 8-9)* **Part 4: Dilations** *(pages 10-11)*

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Part 1: Translations

Pre-Requisites	Basic knowledge of rotations, reflections, and translations (8.G.1)
Standards Addressed	8.G.3: **Describe the effect of** dilations, **translations,** rotations, and reflections **on two-dimensional figures using coordinates.**
Mathematical Practice(s)	MP 2: Reason abstractly and quantitatively MP 3: Construct viable arguments and critique the reasoning of others MP 7: Look for and make use of structure
Materials	Investigating Transformations Using Coordinates– *page 1* (1 copy per student) Transformations Using Coordinates Summary – *page 4*(1 per student) Colored pencils (optional)

Introduce:

- Tell participants that we are now going to look at translations and how the coordinates are affected.

Activity:

- Distribute the Transformations Using Coordinates Worksheet. Have them title it "Translations."
- Have participants observe that Figure #1 is located on the coordinate grid and the coordinates of its vertices are labeled.
- Tell participants that we are going to add 4 to each of the x-coordinates in the original figure. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants add 4 to each of the x-coordinates in the original figure. Have them draw the new figure, label it figure #10, and label the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Add 4 to each x-coordinate.	Figure 10 (5,2) (8,6) (5,7)	• The image is 4 spaces to the right of the original. The lines connecting the corresponding vertices are parallel.

- Tell participants that we are going to subtract 2 from each of the y-coordinates in the original figure. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants subtract 2 from each of the y-coordinates in the original figure. Have them draw the new figure, label it figure #11, and label the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.

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- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Subtract 2 from each y-coordinate.	Figure 11 (1,0) (3,4) (1,5)	• The image is 2 spaces down from the original. The lines connecting the corresponding vertices are parallel.

- Tell participants that we are going to add 4 to each x-coordinate and subtract 2 from each of the y-coordinates in the original figure. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants add 4 to each x-coordinate and subtract 2 from each of the y-coordinates in the original figure. Have them draw the new figure, label it figure #12, and label the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Add 4 to each x-coordinate and subtract 2 from each y-coordinate.	Figure 12 (5,0) (7,4) (5,5)	• The image is shifted 2 spaces to the right and 2 units down from the original. The lines connecting the corresponding vertices are parallel.

- Ask: What observations can you make about the effect that translations have on the coordinates of a figure's vertices. (Have students record their observations in the "Translations" portion of the worksheet titled "Transformations Using Coordinates Summary.")

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Part 2: Reflections

Pre-Requisites	Basic knowledge of rotations, reflections, and translations (8.G.1)
Standards Addressed	8.G.3: **Describe the effect of** dilations, translations, rotations, and **reflections on two-dimensional figures using coordinates.**
Mathematical Practice(s)	MP 2: Reason abstractly and quantitatively MP 3: Construct viable arguments and critique the reasoning of others MP 7: Look for and make use of structure
Materials	Investigating Transformations Using Coordinates – *page 1*(2 copies per student) Transformations Using Coordinates Summary – *page 4* (1 per student) Colored pencils (optional)

A. Reflections in the x-axis and y-axis

Introduce:

- Tell participants that we are first going to look at reflections and how the coordinates are affected. We are going to start by looking at reflections in the axes.

Activity:

- Distribute the Transformations Using Coordinates Worksheet. Have them title it "Reflections in the x-axis and y-axis."
- Have participants observe that Figure #1 is located on the coordinate grid and the coordinates of its vertices are labeled.
- Tell participants that we are going to reflect "Figure 1" in the *y*-axis. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure 1" when it is reflected in the y-axis. Have them label the image "Figure 2" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Reflected Figure 1 in the y-axis	Figure 2 (-1,2) (-3,6) (-1,7)	• The coordinates for Figure 2 have the opposite x-value but the y-value is the same. • The lines that connect the corresponding vertices are parallel and are bisected by the y-axis.

- Tell participants that we are now going to reflect "Figure 1" in the *x*-axis. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.

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- Have participants draw the image of "Figure 1" when it is reflected in the x-axis. Have them label the image "Figure 3" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the next row of table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Reflected Figure 1 in the x-axis	Figure 3 (1,-2) (3,-6) (1,-7)	• The coordinates for Figure 3 have the opposite y-value but the x-value is the same. • The lines that connect the vertices are vertical, parallel, and are bisected by the x-axis.

- Tell participants that we are now going to reflect "Figure 2" in the *x*-axis. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure 2" when it is reflected in the x-axis. Have them label the image "Figure 4" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the next row of table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 2 (-1,2) (-3,6) (-1,7)	Reflected Figure 2 in the x-axis	Figure 4 (-1,-2) (-3,-6) (-1,-7)	• The coordinates for Figure 4 have the opposite y-value but the x-value is the same as figure 2. • The lines that connect the vertices are vertical, parallel, and are bisected by the x-axis.

- Ask: How are the coordinates of Figure 4 related to the coordinates of Figure 1? Why? (Have students record their conclusions in the "Additional Observations" section of their worksheet.)

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B. Reflections in lines (not axes)

Introduce:

- Tell participants that we are going to continue to examine how coordinates are affected by reflections, but this time we are going to reflect the figure in lines that are not the x-axis or y-axis.

Activity:

- Distribute a new copy of the Transformations Using Coordinates Worksheet. Have them title it: "Reflections in lines (not axes)."
- Have participants observe that Figure #1 is located on the coordinate grid and the coordinates of its vertices are labeled.
- Have participants draw the line y = x and label it.
- Tell participants that we are going to reflect "Figure 1" in the line y = x. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.

- Have participants draw the image of "Figure 1" when it is reflected in the line y = x. Have them label the image "Figure 4" and the vertices with coordinates. *Note: If students are having trouble knowing where to place "Figure 4," have them fold their paper to the back along the line y = x. Have them hold the paper up to the light so that they can vaguely see the triangle through the paper and then draw in the triangle.*
- Discuss: How do you know where the image of each vertex is when you reflect in the line y = x?
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Reflected Figure 1 in the line y = x.	Figure 4 (2,1) (6,3) (7,1)	• For figure 4, the x and y coordinates are reversed. All of the values are still positive. • The lines that connect the corresponding vertices are parallel and are bisected by the line y-x.

- Have participants draw the line x = -2 and label it.
- Tell participants that we are going to reflect "Figure 1" in the line x = -2. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure 1" when it is reflected in the line x = -2. Have them label the image "Figure 5" and the vertices with coordinates.
- Discuss: How do you know where the image of each vertex is when you reflect in the line x = -2.
- Have participants connect the corresponding vertices with lines.

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- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Reflected Figure 1 in the line x=-2.	Figure 5 (-5,-2) (-7,6) (-5,-7)	• The coordinates for Figure 5 have the same y-value as figure 1. • The lines that connect the vertices are horizontal, parallel, and are bisected by the line x = -2. • The corresponding vertices are the same distance from the line x = -2.

- Have the participants draw the line y = 1 and label it.
- Tell participants that we are now going to reflect "Figure 1" in the line y = 1. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure 1" when it is reflected in the line y = 1. Have them label the image "Figure 6" and the vertices with coordinates.
- Discuss: How do you know where the image of each vertex is when you reflect in the line y = 1.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the first row of table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Reflected Figure 1 in the line y – 1.	Figure 6 (1,0) (3,-4) (1,-5)	• The coordinates for Figure 6 have the same x-value as figure 1. • The lines that connect the vertices are vertical, parallel, and are bisected by the x-axis. • The corresponding vertices are the same distance from the line y = 1.

- Ask: What observations can you make about the effect that reflections in a line have on the coordinates of a figure's vertices. (Have students record their observations in the "Reflections" portion of the worksheet titled "Transformations Using Coordinates Summary."

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Part 3: Rotations

Pre-Requisites	Basic knowledge of rotations, reflections, and translations (8.G.1)
Standards Addressed	8.G.3: **Describe the effect of** dilations, translations, **rotations**, and reflections **on two-dimensional figures using coordinates.**
Mathematical Practice(s)	MP 2: Reason abstractly and quantitatively MP 3: Construct viable arguments and critique the reasoning of others MP 7: Look for and make use of structure
Materials	Investigating Transformations Using Coordinates – *page 1* (1 copy per student) Transformations Using Coordinates Summary – *page 4* (1 per student) Colored pencils (optional)

Introduce:

- Tell participants that we are now going to look at rotations and how the coordinates are affected.

Activity:

- Distribute the Transformations Using Coordinates Worksheet. Have them title it "Rotations."
- Have participants observe that Figure #1 is located on the coordinate grid and the coordinates of its vertices are labeled.
- Have participants find the origin and label it "Point O."
- Tell participants that we are going to rotate "Figure #1" 90 degrees clockwise using the origin as the center of rotation. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure #1" when it is rotated 90 degrees clockwise using the origin as the center of rotation. Have them label the image "Figure #7" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Rotated Figure #1 90 degrees clockwise about the origin.	Figure 7 (2,-1) (6,-3) (7,-1)	• The opposite of the x-coordinate becomes the y-coordinate. • The y-coordinate becomes the x-coordinate. • The lines between the corresponding vertices are not parallel.

- Tell participants that we are going to rotate "Figure #1" 90 degrees counter-clockwise using the origin as the center of rotation. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.

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- Have participants draw the image of "Figure #1" when it is rotated 90 degrees counter-clockwise using the origin as the center of rotation. Have them label the image "Figure #8" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Rotated Figure 1 90 degrees counter-clockwise about the origin.	Figure 8 (-2,1) (-6,3) (-7,1)	• The opposite of the y-coordinate becomes the x-coordinate. • The x-coordinate becomes the y-coordinate.

- Tell participants that we are going to rotate "Figure #1" 180 degrees using the origin as the center of rotation. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants draw the image of "Figure #1" when it is rotated 180 degrees using the origin as the center of rotation. Have them label the image "Figure #9" and the vertices with coordinates.
- Have participants connect the corresponding vertices with lines.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure 1 (1,2) (3,6) (1,7)	Rotated Figure 1 180 degrees about the origin.	Figure 9 (-1,-2) (-3,-6) (-1,-7)	• Each coordinate becomes it's opposite. • The lines between the corresponding vertices are not parallel.

- Ask: What are the similarities and differences between the effects of reflections (Part 1) and the effects of Rotations (Part 2)? Have participants record their responses in the space for "Additional Observations."
- Ask: What observations can you make about the effect that rotations of 90 degrees clockwise, 180 degrees, and 90 degrees counter-clockwise have on the coordinates of a figure's vertices. (Have students record their observations in the "Rotations" portion of the worksheet titled "Transformations Using Coordinates Summary.")

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Part 4: Dilations

Pre-Requisites	Basic knowledge of rotations, reflections, and translations (8.G.1)
Standards Addressed	8.G.3: **Describe the effect of dilations**, translations, rotations, and reflections **on two-dimensional figures using coordinates.**
Mathematical Practice(s)	MP 2: Reason abstractly and quantitatively MP 3: Construct viable arguments and critique the reasoning of others MP 7: Look for and make use of structure
Materials	Dilations Using Coordinates Worksheets –*pages 2-3* (1 copy per student) Transformations Using Coordinates Summary – *page 4*(1 per student) Extra graph paper (optional) Colored pencils (optional)

Introduce:

- Tell participants that we are now going to look at dilations and how the coordinates are affected.

Activity:

- Distribute the Dilations Using Coordinates Worksheet.
- Have participants observe that Figure A is located on the coordinate grid and the coordinates of its vertices are labeled.
- Tell participants that we are going to now multiply each coordinate in the original figure by 2. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants multiply each coordinate in the original figure by 2. Have them draw the new figure, label it figure B, and label the vertices with coordinates.
- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure A (2,4) (6,4) (6,6) (4,6)	Multiply each coordinate in Figure 1 by 2.	Figure B (4,8) (12,8) (12,12) (8,12)	• The image is larger than the original. • It is shifted up and to the right.

- Tell participants that we are going to now multiply each coordinate in the original figure by 1/2. Ask them to predict what they think will happen to the coordinates. Have them share their prediction with their partner.
- Have participants multiply each coordinate in the original figure by 1/2. Have them draw the new figure, label it figure C, and label the vertices with coordinates.

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- Have participants complete the table (see below). Make sure they list the vertices in the same order as their corresponding vertices.

Coordinates of Original Figure	Explanation of Transformation	Coordinates of Image	Possible Observations
Figure A (2,4) (6,4) (6,6) (4,6)	Multiply each coordinate in Figure 1 by 1/2.	Figure C (1,2) (3,2) (3,3) (2,3)	• The image is smaller than the original • It is shifted down and to the left.

- Choose a vertex from the original figure and connect it to the corresponding vertices on figure B and figure C. Do this for another set of vertices. What do you observe?
- How do the lengths of the sides and the areas of the shapes compare when the coordinates are multiplied by 2? Multiplied by ½? What do you predict would happen if the coordinates were multiplied by 3?
- Discuss: What happens if you multiply the coordinates by a negative number? *Have students multiply the coordinates by a negative number such as -1/2, -1, -1 1/2 or -2. Have them record it on the grid.*
- Additional Questions: Repeat the same process with a new figure drawn so that one of the vertices is the origin? Multiply the coordinates by a whole number greater than 1? Multiply the coordinates by a number between 0 and 1? How do the figures compare?
- Ask: What observations can you make about the effect that dilations have on the coordinates of a figure's vertices. (Have students record their observations in the "Dilations" portion of the worksheet titled "Transformations Using Coordinates Summary

Name: _____ Date: _____ Period: ____